

ESTIMATION OF CORE LOSS IN TRANSFORMER BY USING FINITE ELEMENT  
METHOD

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This thesis is submitted as partial fulfillment of the requirements for the award of the  
Bachelor of Electrical Engineering (Hons.) (Power System)

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June 2012

## **ABSTRACT**

Transformer is one of the important device that always been used in industry. Some of the weakness of using transformer is cost by some losses in core. By solving this problem, it can save more energy usage. To calculate the losses in traditional techniques, it can use nonlinear programming, numerical method and others. Other than that, Finite Element Method (FEM) can be used. It can predict the period for transformer; calculate core losses, flux distribution and others. In this project, FEM technique is applied to calculate the flux and loss distributions in single phase transformer using MATLAB software. This also presented the localized flux density and loss over a core. As the result, the software must be agreed with the experimental data.

## **ABSTRAK**

Pemboleh ubah voltan adalah salah satu alat yang penting yang selalu digunakan di dalam kilang. Salah satu kelemahan ketika menggunakan pembolehubah voltan ini adalah masalah kewangan disebabkan ada sedikit pembaziran di dalam teras pemboleh ubah. Dengan menyelesaikan masalah ini, ia dapat menjimatkan lebih banyak tenaga yang digunakan. Untuk mengira pembaziran menggunakan cara lama boleh digunakan teknik “nonlinear programming”, “numerical method” dan lain-lain teknik lagi. Selain dari itu, “Finite Element Method (FEM)” juga boleh digunakan. FEM boleh menjangka tempoh hayat sesuatu pembolehubah, mengira pembaziran di teras, penyebaran medan magnet dan lain-lain. Di dalam projek ini, teknik FEM digunakan untuk mengira medan magnet dan pembaziran di dalam pemboleh ubah satu fasa dengan menggunakan sistem MATLAB. Ia juga boleh mengesan kepadatan medan magnet dan pembaziran keatas teras. Sebagai kesimpulan, sistem MATLAB yang digunakan mestilah mengikuti dengan data semasa membuat eksperimen.

## TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	TOPIC	i
	DECLARATION	ii
	DEDICATION	iv
	ACKNOWLEDGEMENT	v
	ABSTRACT	vi
	ABSTRAK	vii
	TABLE OF CONTENTS	viii
	LIST OF TABLES	xii
	LIST OF FIGURES	xiii
1	INTRODUCTION	
	1.1 Background	1
	1.2 Project Objective	4
	1.3 Project Scope	4
	1.4 Thesis Outline	5

**LITERATURE REVIEW**

2.1	Introduction	6
2.2	Estimation of Transformer Core Losses using Finite Element Method	7
2.2.1	Principle	8
2.2.2	B-H Curve	9
2.3	Calculation of transformer losses under Non-sinusoidal current using two analytical methods and finite element analysis	11
2.3.1	Transformer losses under Non-sinusoidal currents	11
2.3.2	No Load Loss	12
2.3.3	Load Loss	13
2.3.4	Ohmic Loss	14
2.3.5	Eddy Current loss in winding	14

**TRANSFORMER AND FINITE ELEMENT METHOD**

3.1	Introduction	16
3.2	Transformer	17
3.2.1	History	17
3.2.2	Induction Coils	18
3.2.3	Electric Power Distribution	19
3.2.4	Basic Principle	21
3.2.5	Induction Law	22

3.2.6	Ideal Power Equation	24
3.2.7	Detail Operation	25
3.2.8	Energy Loss	26
	3.2.8.1 Winding Resistance	28
	3.2.8.2 Hysteresis Losses	28
	3.2.8.3 Eddy Current	29
	3.2.8.4 Megnetostriktion	29
	3.2.8.5 Mechanical Loss	30
	3.2.8.6 Stray Loss	30
3.2.9	Permeability	31
3.3	Finite Element Method	34
	3.3.1 Introduction	34
	3.3.2 Finite Element Discretization	35
	3.3.3 Element-Governing Equation	37
	3.3.4 Assembling all the Element	44
	3.3.5 Solving the Resulting Equation	48

## 4

## RESULT AND ANALYSIS

4.1	Introduction	51
4.2	Transformer Used	52
4.3	Transformer Test	53
4.4	Simulation Result	55

**CONCLUSION AND SUGGESTION**

5.1	Introduction	59
5.2	Conclusion	60
5.3	Suggestion	60

**REFERENCES**

**APPENDICES**

## **LIST OF TABLES**

<b>Number of Tables</b>	<b>Name of tables</b>	<b>Page number</b>
3.1	Core Permeability	33
4.1	Parameters to be used in simulation	52



## LIST OF FIGURES

Number of Figures	Name of Figures	Page number
2.1	Hysteresis Loop	9
3.1	Faraday's experiment with induction between coils of wire	18
3.2	Faraday's ring transformer	19
3.3	An ideal transformer	22
3.4	The ideal transformer as a circuit element	24
3.5	Finite element subdivision	35
3.6	Typical Triangular Element	37
3.7	Shape function $\alpha_1$ and $\alpha_2$ for triangular element	39
3.8	Assembly of three elements	45
4.1	Transformer construction	52
4.2	Open circuit test	54
4.3	Transformer losses test	55
4.4	Current flow	56
4.5	Mesh analysis	56
4.6	Heat of the transformer	57
4.7	3D view heat transformer	57
4.8	Increasing Mesh	58

# **CHAPTER 1**

## **INTRODUCTION**

### **1.1 Background**

Transformer is one of the important device that always been used in industry and distribution system. It is a device that transfer electrical potential from one circuit to others circuit using inductor it have in transformer core. Current at primary circuit will flow through its primary coil and create a magnetic flux that will become magnetic field that flow to secondary coil. The magnetic field that has been induced at secondary circuit is electromotive field or voltage.

The effect that has happen at secondary and primary circuit is called inductive coupling. If it has a load that is connected to the secondary circuit, a current will flow through the secondary winding and electrical energy will be transferred from the primary circuit through the transformer before flow to the load.

In an ideal transformer, the induced voltage in the secondary winding is in proportion to the primary voltage and is given by the ratio of the number of turns in the secondary to the number of turns in the primary.

By appropriate selection the ratio of turns, a transformer will make an alternating current (AC) voltage to be stepped up by making secondary turn ( $N_S$ ) greater than primary turn ( $N_P$ ) or it also can be stepped down by making secondary turn ( $N_S$ ) less than primary turn ( $N_P$ ). The windings are coils that will be wound around a ferromagnetic core, air-core transformers being a famous exception.

Some of the weakness of using transformer is cost by some losses in core. By solving this problem, it can save more energy usage. To calculate the losses in traditional techniques, it can use nonlinear programming, numerical method and others. Other than that, finite element method (FEM) can be used. It can predict the period for transformer; calculate core losses, flux distribution and others. In this project, FEM technique is applied to calculate the flux and loss distributions in single phase transformer using Matlab software. This also presented the localized flux density and loss over a core. As the result, the software must be agreed with the experimental data.

Transformer come from word transform is using to change voltage, current or potential by using magnetic field without change it frequency. It can be either step-up or step-down. The use of transformer is quietly often. It can be found from small part to the biggest part of electrical and electronics equipment. As we can see the transformer is use at computer, transmission line, television and others. It is not reliable in energy saving because it has produce heat. In transformer, it has two types of losses that are iron losses and copper losses. An iron loss is happening in core parameters and a copper loss is occurring in winding resistance.

Finite element analysis (FEA) is one of several numerical methods that can be used to solve complex problems and is the dominant method used today. FEA consist five methods that are:

1. Finite Element Method (FEM)
2. Boundary Element Method (BEM)
3. Finite Difference Method (FDM)
4. Moments Method (MM)
5. Monte Carlo Method (MCM)

From these five methods, finite element method (FEM) has been chosen because it can calculate object with any types of shape. FEM is a mathematical method for solving ordinary and elliptic partial differential equation. It can use to calculate object with linear or nonlinear. FEM is useful to obtain an accurate characterization of electromagnetic behavior or magnetic components such as transformers.

## **1.2 Project Objectives**

The aim of this project is to make an analysis from the chosen transformer. From it, the energy that draws from the transformer can be estimate. Some maintenance could be made before it reaches the due time of expired. By doing that, it can save money to buy a new one if the transformer is damage. Other than that, the flux distribution and loss in transformer core can be understood. It also can give some prediction about the period of a transformer by calculate it losses.

## **1.3 Project Scope**

In order to achieve the objectives of the project, the scope of the project are summarized as follow:

- Find a suitable transformer to use in this project
- Make an open circuit test to the transformer.
- Calculate all the parameters that have in the transformer.
- Design the transformer using MATLAB by use PDE Toolbox to see in 3D view

## **1.4 Thesis Outline**

The thesis consists of five chapters. Each one will be elaborated in great details. Chapter 1 describes the overview of the project. This chapter also discusses on objectives, scope, and thesis organization.

Literature review is discussed in Chapter 2. All the past research and reference will be discussed briefly in this chapter. Chapter 3 discusses the details of calculation, transformer parameters and graph.

Chapter 4 discusses on the finite element method (FEM) and transformer. The discussion is based on the calculation and equation that have in FEM also information about transformer. Chapter 5 describes the result and discussion. The MATLAB result will discuss in this chapter.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

In this chapter, the previous research that has been made by others researcher about transformer core losses will be discussed. This literature review will give some ideas and information about how to start and how to construct the work flow in order to complete this final project.

## **2.2 Estimation of Transformer Core Losses using Finite Element Method**

The use of finite element method (FEM) for transformer design and analysis has been proven as a very powerful tool over recent years. It describes a numerical solution to use 2D finite element method to accurately calculate the flux distribution and total core losses in single phase transformer using software. It also presents the localized flux density and loss over a core. This computational result of core loss agrees with experimental result obtained by us in machine laboratory. [13]

For the purpose, modeling of just the transformer is adequate. Therefore an appropriate model of the transformer is defined considering the construction and position of the coils and the current density of them and permeability of transformer coil. Then this model is divided into triangular elements.

By using magnetostatic analysis of the finite element method, the magnetic vector potential of three nodes of each triangular element is calculated and therefore the flux distribution over the model is obtained. Then, the flux density of each element is evaluated because the magnetic vector potential of each element is considered as a linear function of  $x$  and  $y$ . Then the flux density of each element becomes a constant value.



### 2.2.1 Principle

Finite element method (FEM) is a numerical technique for obtaining approximation solution to boundary the value problems of mathematical physics. Especially it has become a very important tool solve electromagnetic problems because of its ability to model geometrically and compositionally complex problems.

Using finite element method (FEM) to solve problems involves three steps. First, the consist of meshing the problem space into contiguous elements of the suitable geometry and assigning appropriate value of the material parameter that are conductivity, permeability and permittivity to each elements. Secondary, the model has to be excited, so that the initial conditions are set up. Finally, the values of the potentials are suitable constrained at the limits of the problem space. The finite element method has the advantage of the geometrical flexibility. It is possible to include a greater density of elements in regions where fields and geometry are rapidly.

The ampere law states that:

$$\nabla \times H = J \quad (2.1)$$

Where,

H: Magnetic field intensity

J: Total current density

### 2.2.2 B-H curve

In transformer analysis, because of ferromagnetic materials properties usually the problems appear in nonlinear form. Magnetic permeability  $\mu=B/H$  is not constant and is a function of magnetic field in each mesh. Therefore the S matrix in equation  $\nabla \times H = J$  is not constant. It is a function of magnetic permeability or magnetic field in each mesh.

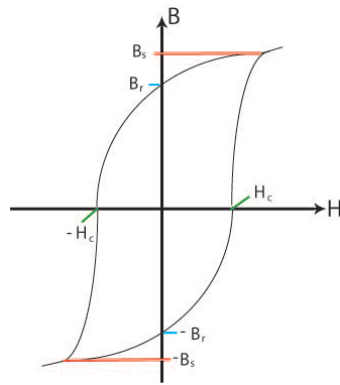


Figure 2.1: Hysteresis Loop

The B-H curve of a ferromagnetic core is a hysteresis loop like Figure 1. The upper approximation of hysteresis loop can be used for calculation of short circuit reactance or radial and axial electromagnetic force on the transformer coils but for calculation of flux distribution and loss in transformer cores, the B-H loop is used. For single phase transformer in nominal circumstance, the no load current and voltage can be measured by using digital scope. Nominal voltage of primary winding, the value of B and H can be calculated from the following equation:

$$e(t) = V(t) - Ri_o = N \frac{d\phi}{dt} \quad (2.2)$$

$$\phi = \frac{1}{N} \int e(t) dt \quad (2.3)$$

$$H = \frac{Ni}{L} \quad (2.4)$$

Where,

$i_o$ : no load current

$V(t)$ : Terminal voltage in no load circumstance

$E(t)$ : EMF

$\phi$ : Flux

$R$ : Resistance of winding

$N$ : Number of turn

$L$ : Mean length

The actual B-H loop of transformer core which is accessed from experiment is used. Simulation algorithm of hysteresis loop is like flow chart. By using third order equation, permeability of each part can be calculated as a function of B ( $\mu=f(B)$ ). Core treatment can be well predicted by using the suggested third order equation model. Calculation results accessed in FEM shows that by the model of core presented in this paper we can estimate core loss with high accuracy and flux distribution in the core can determine locally. We also can find hot spots inside the core. Calculations show that as the number of used meshes is increased the more exact result is accessed. The modeling that is shown in this paper allows us to know the transformer behavior before manufacturing them and thus reducing the design time and cost

## **2.3 Calculation of transformer losses under Non-sinusoidal current using two analytical methods and finite element analysis**

The effectual parameters on the loss of a single phase transformer under harmonic condition have been evaluated. Then, power reduction rate, maximum permissible current and also transformer losses have been considered and calculated by analytic that are using IEEE C57.110 standard and corrected harmonic loss factor and finite element method (FEM). FEM has been used as a very precise method for calculating the loss of the transformer under non-sinusoidal current.

### **2.3.1 Transformer losses under Non-sinusoidal currents**

Transformer manufacturer usually try to design transformers in a way that their minimum losses occur in rated voltage, rated frequency and sinusoidal current. However by increasing the number of non linear load in recent years, the load current is no longer sinusoidal. This non sinusoidal current causes extra loss and increasing temperature in transformer.

Transformer loss is divided into two major groups that are no load and load loss. As a following:

$$P_T = P_{NL} + P_{LL} \quad (2.5)$$

$P_T$ : Total loss in transformer

$P_{NL}$ : No load loss

$P_{LL}$ : Load loss

A brief description of transformer losses and harmonic effects on them is presented in following:

1. No load loss
2. Load Loss
3. Ohmic Loss
4. Eddy current loss in windings

### **2.3.2 No load loss**

No load loss or core loss appears because of time variable nature of electromagnetic flux passing through the core and its arrangement is affected the amount of this loss. Since distribution transformers are always under service, considering the number of this type of transformer in network, the amount of no load loss is high but constant. This type of loss is caused by hysteresis phenomenon and eddy currents into the core. These losses are proportional to frequency and maximum flux density of the core and separated from load current.

Many experiments have shown that core temperature increase is not a limiting parameter in determination of transformers permissible current in the non sinusoidal current. Furthermore, considering that the value of voltage harmonic component is less than 5% only the main component of the voltage is considered to calculate no load loss, the error of ignoring the harmonic component is negligible. So IEEE C57.110 standard has not considered the core loss increase due to non linear loads and has supposed this loss constant, under non sinusoidal currents.

### 2.3.3 Load Loss

Load loss includes DC or ohmic loss, eddy loss in winding and other stray loss and it can be obtained from short circuit test:

$$P_{LL} = P_{DC} + P_{EC} + P_{OSL} \quad (2.6)$$

In above equation

$P_{DC}$ : Loss due to resistance of windings

$P_{EC}$ : Windings eddy current loss

$P_{OSL}$ : The other stray loss in structural parts of transformer such as tank, clamps and others.

The sum of  $P_{EC}$  and  $P_{OSL}$  is called total stray loss. We can calculate the value from the difference of load loss and ohmic loss:

$$P_{TSL} = P_{EC} + P_{OSL} = P_{LL} - P_{DC} \quad (2.7)$$

It should be mentioned that there is no practical or experimental process to separate windings eddy loss and other stray loss yet.

#### **2.3.4 Ohmic Loss**

The loss can be calculated by measuring winding DC resistance and load current. If RMS value of load current increases due to harmonic component, this loss will increase by square of RMS of load current. The windings ohmic loss under harmonic condition is shown:

$$P_{dc} = R_{dc} I^2 = R_{dc} \sum_{h=1}^{h=h_{max}} I_{h,max}^2 \quad (2.8)$$

#### **2.3.5 Eddy current loss in windings**

This loss is caused by time variable electromagnetic flux that covers windings. Skin effect and proximity effect are the most important phenomenon in creating these losses.

Also, the most amount of loss is in the last layer of conductors in winding, which is due to high radial flux density in this region

$$P_{EC} = \frac{\pi \tau^2 \mu^2}{3\rho} f^2 x H^2 \propto f^2 x I^2 \quad (2.9)$$

In this equation

$\tau$ : A conductor width perpendicular to field line

$\rho$ : Conductor's resistance



## 3.2 Transformer

Transformers range of size can be found from a thumbnail-sized coupling transformer that is hidden inside a stage microphone to huge units that have weighing hundreds of tons that is been used to interconnect portions of power grids. All of these transformers are operate on the same basic principles even though the range of their designs is many. While, for new technologies that have eliminated the need of transformers in some electronic circuits, transformers are still found in nearly all electronic devices designed for household alternating current (AC). Transformers are essential for high-voltage electric power transmission, which makes long-distance transmission economically practical.

### 3.2.1 History

The principle behind the operation of a transformer, electromagnetic induction, was discovered independently by Michael Faraday and Joseph Henry in 1831. However, Faraday was the first to publish the results of his experiments and thus receive credit for the discovery. The relationship between electromotive force (EMF) or voltage and magnetic flux was formalized in an equation now referred to as Faraday's law of induction.

$$|\varepsilon| = \left| \frac{d\phi_B}{dt} \right| \quad (3.1)$$

Where  $|\varepsilon|$  the magnitude of the EMF in volts and  $\phi_B$  is the magnetic flux through the circuit in Weber. Faraday performed the first experiments on induction

between coils of wire, including winding a pair of coils around an iron ring, therefore creating the first toroidal closed-core transformer. However he only applied individual pulses of current to his transformer, and never discovered the relation between the turns ratio and EMF in the windings. [14]

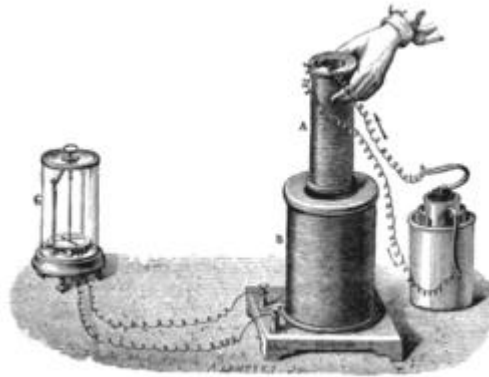


Figure 3.1: Faraday's experiment with induction between coils of wire [14]

### 3.2.2 Induction coils

The first type of transformer to see wide use was the induction coil, invented by Rev. Nicholas Callan of Maynooth College, Ireland in 1836. He was one of the first researchers to realize that the more turns the secondary winding has in relation to the primary winding, the larger is the increase in EMF. Induction coils evolved from scientists' and inventors' efforts to get higher voltages from batteries. Since batteries produce direct current (DC) rather than alternating current (AC), induction coils relied upon vibrating electrical contacts that regularly interrupted the current in the primary to create the flux changes necessary for induction. Between the 1830s and the 1870s, efforts to build better induction coils, mostly by trial and error, slowly revealed the basic principles of transformers. [14]

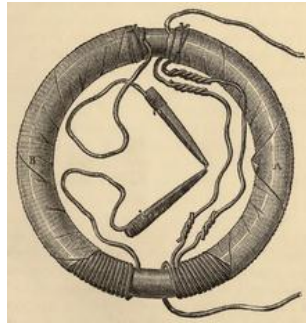


Figure 3.2: Faraday's ring transformer [14]

### 3.2.3 Electric Power Distribution

Before the development of the transformer, electric power distribution primarily used direct current. It was difficult for a DC utility-power generation station to be more than a few kilometers from the user, because up until about 1897, light bulbs could only be effectively designed to operate at up to 110 volts maximum and up to 220 volts by 1917. It is expensive to send energy to long distances at utility voltage that are 100-250 volts due to the very high amperage of many customers and the need for very thick transmission wires capable of handling the current.

It was understood that high voltages allowed long distance transmission with low amperage it is 250 volts at 5000 amps that is equal to 25000 volts at 50 amps so the transmission wires can be smaller and less expensive, but it still needed to be stepped back down to utility voltage at the customer's location.

At the time the only way to efficiently convert DC from one voltage to another was with a spinning motor-generator device, and this would be needed at each customer site. Each motor-generator has brushes constantly rubbing on a commutator, and axle bearings that need lubrication. The brushes wear out and need to be periodically replaced and the commutator wears down and needs to be resurfaced, then the whole machine is rebuilt when the commutator wears too thin.

By the 1870s, efficient generators that produced alternating current were available, and it was found that alternating current could power an induction coil directly without an interrupter. In 1876, Russian engineer Pavel Yablochkov invented a lighting system based on a set of induction coils where the primary windings were connected to a source of alternating current and the secondary windings could be connected to several electric candles that are arc lamps that is his own design. The coils Yablochkov employed functioned essentially as transformers.

In 1878, the Ganz Company in Hungary began manufacturing equipment for electric lighting and, by 1883, had installed over fifty systems in Austria-Hungary. Their systems used alternating current exclusively and included those comprising both arc and incandescent lamps, along with generators and other equipment.

Lucien Gaulard and John Dixon Gibbs first exhibited a device with an open iron core called a secondary generator in London in 1882, then sold the idea to the Westinghouse company in the United States. They also exhibited the invention in Turin, Italy in 1884, where it was adopted for an electric lighting system. However, the efficiency of their open-core bipolar apparatus remained very low.

Induction coils with open magnetic circuits are inefficient for transfer of power to loads. Until about 1880, the paradigm for AC power transmission from a high voltage supply to a low voltage load was a series circuit. Open-core transformers with a ratio near 1:1 were connected with their primaries in series to allow use of a high voltage for